Business Application of Matrices:

Example: A manufacturer produces three products A, B and C and sells these products in two markets. Number of units of annual sales of these products in the two markets are given below.

	Products A	B	C
Market I	10,000	2,000	8.000
Market II	6,000	20,000	4,000

i) if the unit price of A, B and C are Rs 25, Rs 12 and Rs 15 respectively, find the total revenue in each market.

ii) if the unit cost of products A, B and C are Rs. 18, Rs 10 and Rs 8 respectively, find the gross profit in each market. (solve by using matrix)

Solution:

Let the Quantity matrix, $Q = \begin{bmatrix} 10.000 & 2.000 & 8000 \\ 6,000 & 20.000 & 4000 \end{bmatrix}$

Price matrix, $P = \begin{bmatrix} 25 \\ 12 \\ 15 \end{bmatrix}$

and cost matrix, $C = \begin{bmatrix} 18 \\ 10 \\ 8 \end{bmatrix}$

i)

$$\therefore Q.P = \begin{bmatrix} 10,000 & 2.000 & 8.000 \\ 6.000 & 20,000 & 4,000 \end{bmatrix} \times \begin{bmatrix} 25 \\ 12 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} (10\,000 \times 25) + (2,000 \times 12) + (8\,000 \times 15) \\ (6000 \times 25) + (20,000 \times 12) + (4.000 \times 15) \end{bmatrix}$$

$$= \begin{bmatrix} 250000 + 24000 + 120000 \\ 150\,000 + 240000 + 60\,000 \end{bmatrix}$$

$$= \begin{bmatrix} 3,94000 \\ 4,50000 \end{bmatrix}$$

$\therefore$ The total revenue earned in market I is Rs. 3,94000 and total revenue earned in market II is Rs. 4,50,000.

ii) Again

$$Q.C = \begin{bmatrix} 10{,}000 & 2{,}000 & 8{,}000 \\ 6{,}000 & 20{,}000 & 4{,}000 \end{bmatrix} \times \begin{bmatrix} 1.8 \\ 10 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} (10{,}000\times18)+(2{,}000\times10)+(8000\times8) \\ (6000\times18)+(20000\times10)+(4000\times8) \end{bmatrix}$$

$$= \begin{bmatrix} 180000+20000+64000 \\ 108000+200000+32000 \end{bmatrix}$$

$$= \begin{bmatrix} 264000 \\ 340000 \end{bmatrix}$$

∴ The total cost of Product sold in the market I and market II are Rs. 2,64,000 and Rs 3,40,000 respectively.

Required gross profit in market I = Rs. 394000 − Rs 264000
= Rs. 1,30,000

& gross Profit in market II = Rs 4,50,000 − Rs 3,40,000
= 1,10,000

Example: A company is considering its three methods of production in producing three goods A, B and C. The amount of each good A, B and C produced by each method is shown in the following matrix:

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \text{Methods} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 4 & 8 & 2 \\ 5 & 7 & 1 \\ 5 & 3 & 9 \end{bmatrix} \end{array}$$

The column matrix $\begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$ represents the profit per unit for A, B and C in order. Using matrix multiplication find which method maximizes the total profit. (G.U. 2007)

Solution: Let $Q = \begin{bmatrix} 4 & 8 & 2 \\ 5 & 7 & 1 \\ 5 & 3 & 9 \end{bmatrix}$, $P = \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$

$$\therefore \quad Q.P = \begin{bmatrix} 4 & 8 & 2 \\ 5 & 7 & 1 \\ 5 & 3 & 9 \end{bmatrix} \times \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} (4\times10)+(8\times4)+(2\times6) \\ (5\times10)+(7\times4)+(1\times6) \\ (5\times10)+(3\times4)+(9\times6) \end{bmatrix}$$

$$= \begin{bmatrix} 40+32+12 \\ 50+28+6 \\ 50+12+54 \end{bmatrix} = \begin{bmatrix} 84 \\ 84 \\ 116 \end{bmatrix}$$

∴ The total profit gaind by the method 1, 2 and 3 are 84, 84 and 116 respectively.

∴ The method 3 maximizes the total profit.

Example: In a certain city there are 50 colleges and 400 schools. Each college and school has 18 peons, 5 clerks and 1 cashier. Each college in addition has 1 section officer and 1 Librarian. The monthly salary of each of them is as follows -

Peon = Rs 300, clerk = Rs 500, cashier = Rs 600, Section officer = Rs 700 and Librarian = Rs 900

use matrix method find

a) Total no. of posts of each kind in colleges and schools taken together

b) The total monthly salary bill of all the colleges and schools taken together.

Solution:

Let $A = \begin{bmatrix} 50 & 400 \end{bmatrix}$, represents the no. of colleges and schools respectively.

$B = \begin{bmatrix} 18 & 5 & 1 & 1 & 1 \\ 18 & 5 & 1 & 0 & 0 \end{bmatrix}$, where column represents the no. of peon, clerk, cashier, section officer and Librarian while the row represents no. of colleges and schools respectively.

and $S = \begin{bmatrix} 300 \\ 500 \\ 600 \\ 700 \\ 900 \end{bmatrix}$, represents the monthly salary of peon, clerk, cashier, section officer and Librarian respectively.

a) Now, $A.B = \begin{bmatrix} 50 & 400 \end{bmatrix} \times \begin{bmatrix} 18 & 5 & 1 & 1 & 1 \\ 18 & 5 & 1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} (50\times18)+(400\times18) & (50\times5)+(400\times5) & (50\times1)+(400\times1) & (50\times1)+(400\times0) & (50\times1)+(400\times0) \end{bmatrix}$

$= \begin{bmatrix} (900+7200) & (250+2000) & (50+400) & (50+0) & (50+0) \end{bmatrix}$

$= \begin{bmatrix} 8100 & 2250 & 450 & 50 & 50 \end{bmatrix}$

$\therefore$ Total no. of Peons = 8100
Total no. of clerks = 2250
Total no. of cashier = 450
Total no. of Section officer = 50
2 Total no. of Librarian = 50

b)

$$B.S = \begin{bmatrix} 18 & 5 & 1 & 1 & 1 \\ 18 & 5 & 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 300 \\ 500 \\ 600 \\ 700 \\ 900 \end{bmatrix}$$

$$= \begin{bmatrix} (18\times300)+(5\times500)+(1\times600)+(1\times700)+(1\times900) \\ (18\times300)+(5\times500)+(1\times600)+(0\times700)+(0\times900) \end{bmatrix}$$

$$= \begin{bmatrix} 5400+2500+600+700+900 \\ 5400+2500+600+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 10100 \\ 8500 \end{bmatrix}$$

$\therefore$ Total monthly salary bill of each college and school are 101,00 and 8500 respectively.

Again, $A\cdot(BS) = \begin{bmatrix} 50 & 400 \end{bmatrix} \times \begin{bmatrix} 10100 \\ 8500 \end{bmatrix}$

$$= [(50\times10100)+(400\times8500)]$$

$$= [505000+3400000]$$

$$= [3,905,000]$$

$\therefore$ Required total monthly bill of all colleges and schools taken together is 39,05,000

Example: If $P=\begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$, $Q=\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$; find the matrix R such that $5P+3Q+2R$ is a null matrix.

Solution: Let $R=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Now,

$$5P+3Q+2R = 5.\begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix} + 3.\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} + 2.\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 45 & 5 \\ 20 & 15 \end{bmatrix} + \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 2a_{11} & 2a_{12} \\ 2a_{21} & 2a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 45+3+2a_{11} & 5+15+2a_{12} \\ 20+21+2a_{21} & 15+36+2a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 48+2a_{11} & 20+2a_{12} \\ 41+2a_{21} & 51+2a_{22} \end{bmatrix}$$

A/Q, $5P+3Q+2R = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 48+2a_{11} & 20+2a_{12} \\ 41+2a_{21} & 51+2a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore 48+2a_{11}=0$; $20+2a_{12}=0$,

$\Rightarrow 2a_{11}=-48$; $\Rightarrow 2a_{12}=-20$

$\Rightarrow a_{11}=-\frac{48}{2}$; $\Rightarrow a_{12}=-10$

$=-24$

$41+2a_{21}=0$, ; $51+2a_{22}=0$

$\Rightarrow 2a_{21}=-41$; $\Rightarrow 2a_{22}=-51$

$\Rightarrow a_{21}=-\frac{41}{2}$; $\therefore a_{22}=-\frac{51}{2}$

$\therefore$ Required. $R=\begin{bmatrix} -24 & -10 \\ -\frac{41}{2} & -\frac{51}{2} \end{bmatrix}$ //

Example! Show that the matrix $A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the condition $A^2-4A=5I$, where I is the identity matrix of order 3

Solution! $A^2-4A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4.\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} (1\times1+2\times2+2\times2) & (1\times2*2\times1+2\times2) & (1\times2+2\times2*2\times1) \\ (2\times1+1\times2+2\times2) & (2\times2+1\times1+2\times2) & (2\times2+1\times2+2\times1) \\ (2\times1+2\times2+1\times2) & (2\times2+2\times1+1\times2) & (2\times2+2\times2+1\times1) \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (1+4+4) & (2+2+4) & (2+4+2) \\ (2+2+4) & (4+1+4) & (4+2*2) \\ (2+4+2) & (4+2+2) & (4+4+1) \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$$A^2-4A=\begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}-\begin{bmatrix}4\\8\\8\end{bmatrix}$$

$$=\begin{bmatrix}(9-4)&(8-8)&(8-8)\\(8-8)&(9-4)&(8-8)\\(8-8)&(8-8)&(9-4)\end{bmatrix}$$

$$=\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$$

$$=5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=5.I$$

Thus $A^2-4A=5I$