COM-HC-4026 (1)

Business Mathematics

unit-1 Matrices and Determinants

§ Matrices: m×n real numbers arranged in a rectangular way with m rows and n columns enclosed by a pair of brackets [ ] or ( ) is called an m×n matrix

Types of matrices:

Square matrix: A matrix in which the no. of rows is equal to the no. of columns is called a square matrix.

eg. $\begin{bmatrix} 2 & 4 \\ 3 & -7 \end{bmatrix}_{2\times2}$ is a square matrix.

Diagonal element: The elements a_{ij} of the matrix $A=[a_{ij}]_{m\times n}$ for $i=j$ are called diagonal elements of the matrix A.

eg in $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$, the diagonal elements are 1 and 5

Diagonal Matrix: A square matrix in which all non-diagonal elements are equal to zero, is called a diagonal matrix. eg $A=\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}_{3\times3}$, here $a_{11}=a$, $a_{22}=b$, $a_{33}=c$

Scalar matrix: A diagonal matrix whose diagonal elements are equal to each other, is called a scalar matrix. eg $A=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3\times3}$

Unit Matrix: A scalar matrix each of whose diagonal elements is unity (one) is called a unit matrix or an identity matrix and is denoted by I

eg $I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$ is an identity matrix of order 3×3

Null Matrix: If all the elements of a matrix are equal to zero, the matrix is called a zero matrix or a null matrix. It is denoted by O.

eg. $O=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times3}$ is a null matrix of order 2×3

Column Matrix: A matrix containing only a single column is called a column matrix

eg $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$, is a column matrix of order 2×1

Row Matrix: A matrix containing only a single row is called a Row matrix.

eg. $\begin{bmatrix} 2 & 4 & 1 \end{bmatrix}$, is a row matrix of order 1×3

§ **Equality of Matrices**: Two matrices are said to be equal if and only if they are of same order and the elements in the corresponding places of two matrices are equal.

eg. if $A = \begin{bmatrix} 3 & 4 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2^2 \\ 0 & -1 \end{bmatrix}$

Then A and B are equal ie $A = B$

§ **Sum of Matrices**:

a) **Sum of Matrices**: If two matrices $A = (a_{ij})$ and $B = (b_{ij})$ are of same order $m \times n$, then $A + B$ is defined a new matrix $C = (c_{ij})$ of same order $m \times n$ in which $c_{ij} = a_{ij} + b_{ij}$

eg if $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{2\times2}$

Then $A + B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}_{2\times2}$

b) **Scalar multiple of a matrix**: The scalar multiple of a matrix $A = (a_{ij})_{m\times n}$ by a scalar K is the matrix $C = (c_{ij})_{m\times n}$ where $c_{ij} = K a_{ij}$

eg. if $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, Then $K.A = \begin{bmatrix} K.2 & K.3 \\ K.4 & K.5 \end{bmatrix}$

c) **Negative of Matrix**: The negative of a matrix is $-A$, defined as a scalar multiple of A by $-$

d) **Difference of two matrices**: The difference of two matrices A and B is defined as the Sum of A and $-B$

§ **Product of Two Matrices**:

Definition: If A be an $m\times n$ matrix and B be an $n\times$ matrix, then the product of A and B is denoted by AB is the $m\times r$ matrix whose entry in the ith row and jth column is the sum of product of corresponding element of ith row of A and jth column of B.

Product AB is defined only when the no. of columns of A is equal to the no. of rows of B.

eg. if $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}_{2\times2}$ and $B = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 5 \end{bmatrix}_{2\times3}$

Then AB exists, as no. of column of $A = 2 =$ no. of rows

Q If $A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$, Then find $2A + 3B$. (G.U. 2009)

Solution:

$$2A + 3B = 2 \cdot \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} + 3 \cdot \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix} + \begin{bmatrix} 21 & 18 & 9 \\ 3 & 12 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 0+21 & 4+18 & 6+9 \\ 4+3 & 2+12 & 8+15 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 22 & 15 \\ 7 & 14 & 23 \end{bmatrix}$$

Q If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix}$, Then find AB and BA. verify whether $AB = BA$. (G.U. 2009)

Ans: Since the no. of column of A is equal to the no. of rows of B

$\therefore$ AB is defined

Now $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix}$

$$= \begin{bmatrix} (1\times5)+(2\times0) & (1\times6)+2(-2) \\ (3\times5)+(4\times0) & (3\times6)+4.(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 5+0 & 6-4 \\ 15+0 & 18-8 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 15 & 10 \end{bmatrix}$$

Again, The no. of column of B is the equal to the no. of rows of A, Therefore BA is defined.

Now. $BA = \begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} (5\times1)+(6\times3) & 5\times2+6\times4 \\ (0\times1)+(-2)\times3 & 0\times2+(-2)\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 5+18 & 10+24 \\ 0-6 & 0-8 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 34 \\ -6 & -8 \end{bmatrix}$$

$\therefore AB \neq BA$