**LINEAR PROGRAMMING**

Many applications in mathematics involve systems of inequalities/equations. In this note, we shall apply the systems of linear inequalities/equations to solve some real life problems of the type as given below:

A furniture dealer deals in only two items–tables and chairs. He has Rs 50,000 to invest and has storage space of at most 60 pieces. A table costs Rs 2500 and a chair Rs 500. He estimates that from the sale of one table, he can make a profit of Rs 250 and that from the sale of one chair a profit of Rs 75. He wants to know how many tables and chairs he should buy from the available money so as to maximise his total profit, assuming that he can sell all the items which he buys.

Such type of problems which seek to maximise (or, minimise) profit (or, cost) form a general class of problems called **optimisation problems**. Thus, an optimization problem may involve finding maximum profit, minimum cost, or minimum use of resources etc.

A special but a very important class of optimisation problems is **linear programming problem.** The above stated optimisation problem is an example of linear programmingproblem. Linear programming problems are of much interest because of their wideapplicability in industry, commerce, management science etc.

**Linear Programming Problem and its Mathematical Formulation**

We begin our discussion with the above example of furniture dealer which will further lead to a mathematical formulation of the problem in two variables. In this example, we observe:

1. The dealer can invest his money in buying tables or chairs or combination thereof. Further he would earn different profits by following different investment strategies.
2. There are certain **overriding conditions** or **constraints** viz., his investment is limited to a **maximum** of Rs 50,000 and so is his storage space which is for a maximum of 60 pieces.

Suppose he decides to buy tables only and no chairs, so he can buy 50000 ÷ 2500,

i.e., 20 tables. His profit in this case will be Rs (250 × 20), i.e., **Rs 5000.**Suppose he chooses to buy chairs only and no tables. With his capital of Rs 50,000, he can buy 50000 ÷ 500, i.e. 100 chairs. But he can store only 60 pieces. Therefore, he is forced to buy only 60 chairs which will give him a total profit of Rs (60 × 75), i.e., **Rs 4500**.

There are many other possibilities, for instance, he may choose to buy 10 tables and 50 chairs, as he can store only 60 pieces. Total profit in this case would be Rs (10 × 250 + 50 × 75), i.e., **Rs 6250** and so on. We, thus, find that the dealer can invest his money in different ways and he would earn different profits by following different investment strategies.

Now the problem is: How should he invest his money in order to get maximum profit? To answer this question, let us try to formulate the problem mathematically.

**Mathematical formulation of the problem**

Let *x* be the number of tables and *y* be the number of chairs that the dealer buys.

Obviously, *x* and *y* must be non-negative, i.e.

0 ... (1)

(Non-negative constraints)

0 ... (2)

*x*

*y*

The dealer is constrained by the maximum amount he can invest (Here it is Rs 50,000) and by the maximum number of items he can store (Here it is 60).

Stated mathematically,

2500*x* + 500*y* ≤50000 (investment constraint)

or 5*x* + *y* ≤100 ... (3)

and *x* + *y* ≤60 (storage constraint) ... (4)

The dealer wants to invest in such a way so as to maximise his profit, say, Z which

stated as a function of *x* and *y* is given by

Z = 250*x* + 75*y* (called *objective function*) ... (5)

Mathematically, the given problems now reduces to:

Maximise Z = 250*x* + 75*y*

subject to the constraints:

5*x* + *y* ≤100

*x* + *y* ≤60

*x* ≥0, *y* ≥0

So, we have to maximise the linear function Z subject to certain conditions determined by a set of linear inequalities with variables as non-negative. There are also some other problems where we have to minimise a linear function subject to certain conditions determined by a set of linear inequalities with variables as non-negative. Such problems are called **Linear Programming Problems.**

Thus, a Linear Programming Problem is one that is concerned with finding the **optimal value** (maximum or minimum value) of a linear function (called **objective** **function**) of several variables (say *x* and *y*), subject to the conditions that the variables are **non-negative** and satisfy a set of linear inequalities (called **linear constraints).**

The term **linear** implies that all the mathematical relations used in the problem are **linear relations** while the term programming refers to the method of determining a particular **programme** or plan of action.

**Definition of some terms**

1. **Objective function:** Linear function Z = *ax* + *by*, where *a*, *b* are constants, which has to be maximised or minimized is called a linear **objective function.**

In the above example, Z = 250*x* + 75*y* is a linear objective function. Variables *x* and

*y* are called **decision variables**.

1. **Constraints** The linear inequalities or equations or restrictions on the variables of a

linear programming problem are called **constraints**. The conditions *x* ≥0, *y* ≥0 are

called non-negative restrictions. In the above example, the set of inequalities (1) to (4)

are **constraints**.

1. **Optimisation problem** A problem which seeks to maximise or minimise a linear function (say of two variables *x* and *y*) subject to certain constraints as determined by a set of linear inequalities is called an **optimisation problem**. Linear programming problems are special type of optimisation problems. The above problem of investing a given sum by the dealer in purchasing chairs and tables is an example of an optimization problem as well as of a linear programming problem.